Reg. No. : [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

Question Paper Code : 87507

M.C.A. DEGREE EXAMINATION, FEBRUARY 2012.

Second Semester

DMC 1921 — MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE

(Regulation 2009)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State Cayley Hamilton theorem.

2. Write the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$.

3. How many words of three distinct letters can be formed from the letters of the word "MASTER"?

4. If A, B, C are three finite sets, then $[n\,(A \cup B \cup C)] =$ ————————.

5. Verify whether $(P \vee Q) \rightarrow P$ is a tautology.

6. Write in symbolic form "Jack and Jill went up the hill".

7. Write the types of Grammars.

8. When is a Grammar said to be ambiguous?

9. Write the context free Grammar for the language $L = \{a^n b^n / n \geq 1\}$.

10. When are two states said to be equivalent?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the inverse of the matrix $A = \begin{pmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{pmatrix}$. (8)

(ii) Solve : $x_1 + x_2 + 2x_3 = 1$
$2x_1 + x_2 + 4x_3 = 2$
$3x_1 + 5x_2 + x_3 = -1$. (8)

Or

(b) (i) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, find a matrix P so that $P^{-1}AP$ is a diagonal matrix. (8)

(ii) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$. (8)

12. (a) (i) Show that $(A - B) - C = A - (B \cup C)$. (4)

(ii) Show that $A \times (B \cup C) = (A \times B) \cup (A \times C)$. (4)

(iii) In a city, 3 daily news papers X, Y, Z are published, 65% people of the city read X, 54% read Y, 45% read Z, 38% read X and Y, 32% read Y and Z, 28% read X and Z, 12% do not read any of the three papers. If 10,00,000 persons live in the city, find the numbers of persons who read all the three papers. (8)

Or

(b) (i) For any integers n, r such that $0 \le r \le n$, show that $C(n, r) = C(n, n - r)$. (4)

(ii) Generate the Pascal's triangle. (4)

(iii) Let $A = \{1, 2, 3, 4\}$ *and* $R = \{(1, 2)\ (2, 3)\ (3, 3)\ (3, 4)\ (4, 2)\}$ be a relation defined on A. Find the transitive closure of R. (8)

13. (a) (i) Obtain the conjunctive normal form of the formula

$(p \wedge \neg(q \vee r)) \vee (((p \wedge q) \vee \neg r) \wedge p)$. (8)

(ii) Prove the following using CP rule

$(P \vee Q) \rightarrow R \Rightarrow (P \wedge Q) \rightarrow R$. (8)

Or

(b) (i) Check whether $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology or not. (8)

(ii) Find the validity of the following :

$P \rightarrow (\exists x)Q(x) \Leftrightarrow (\exists x)(P \rightarrow Q(x))$. (8)

14. (a) (i) Give the context free grammar to generate the language "the set of strings over the alphabet $\{a, b\}$ with more a's than b's". (8)

(ii) State and prove pumping lemma. (8)

Or

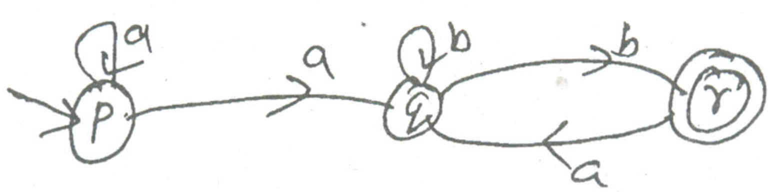
(b) (i) Prove that :

$L = \{a^p / p \text{ is a prime}\}$ is not regular. (8)

(ii) Define regular Grammar and generate the regular grammar to generate "the set of all strings having even number of zero's". (8)

15. (a) (i) Construct an automaton to accept the set of strings with length divisible by 2 or 5 over $\{a, b\}$. (8)

(ii) Convert the following NFA to DFA. (8)



Or

(b) (i) Construct a NFA accepting all strings over $\{0, 1\}$ which end in 1 but does not contain the substring 00. (8)

(ii) Prove that, L be the set accepted by a NFA M, then there exists a FA M' which accepts L. (8)

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